Particles, Feynman diagrams and all that[1]

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Abstract
Quantum fields are introduced in order to give students an accurate qualitative understanding of the origin of Feynman diagrams as representations of particle interactions. Elementary diagrams are combined to produce diagrams representing the main features of the Standard Model.

Introduction

Elementary particles and their interactions in terms of Feynman diagrams now form part of some AS/A2 Physics courses. For example, in the AS Module 1 of the AQA GCE Physics (Specification A) course the candidates are expected to have knowledge and understanding of:

- The concepts of exchange particles to explain forces between elementary particles.
- Simple Feynman diagrams to show how a reaction occurs in terms of particles going in and out and exchange particles: limited to beta minus decay, beta plus decay, electron capture, neutrino–neutron collisions and electron–proton collisions.

Similarly the AS Unit3: Topic 3C of the EDEXCEL GCE Physics course demands knowledge and understanding of:

- Forces described in terms of exchange particles including photons, W^+, W^- and Z particles, and gluons.
- Use of simple Feynman diagrams involving exchange particles.

The objective of this article is to introduce the theory of *quantum fields* and develop it sufficiently to gain an accurate **qualitative** understanding of the origin of Feynman diagrams as representations of particle interactions. We endeavour to keep the use of mathematics to an absolute minimum. We hope this exposition will allow students to develop an understanding of Feynman diagrams that goes beyond the rudimentary drawing and labelling of the diagrams themselves. The article will also help to clarify some misconceptions, which have appeared in certain textbooks, regarding the structure of the diagrams.

[1] This work was carried out when the author was a Teacher Fellow Commoner at Corpus Christi College, Cambridge.

Quantum fields

We are familiar with classical fields, for example the electromagnetic field. A given distribution of electric charges and currents gives rise to electric and magnetic fields. The strengths of these fields at each point in space can vary with time. How the fields vary with time is described by a set of equations, which are known as Maxwell equations. So, a classical field is a quantity that is defined continuously over all space, and this quantity in general varies with time. The time variations are governed by appropriate wave equations. Quantum fields are different. Here a field becomes an operator. For every point in space the field operator can act on vacuum to create states of definite momentum and energy. Out of all possible states there is one with lowest energy. This state can be described in terms of momentum

$\boldsymbol{p}$, which can have all possible magnitudes and directions, and energy E given by

$$E^2 = (c\boldsymbol{p})^2 + (mc^2)^2$$

where c is the speed of light and m is a constant with dimensions of mass. This relationship, however, is identical with the relation in the Special Theory of Relativity between the energy, E, and the momentum, $\boldsymbol{p}$, of a relativistic particle that has rest mass m. Such a state, therefore, can be interpreted as a particle of rest mass m, which has emerged out of the action of an appropriate quantum field operating on the vacuum state.

So particles can emerge from quantum fields, through the process of quantization of classical fields. **Quantization of a classical field is the formal procedure that turns a classical field into an operator capable of creating particles from vacuum**. The quantization of the familiar electromagnetic field gives rise to particle-like quanta, the photons. In nature, apart from the photon, there are other elementary particles, like electrons. We can think of these as the quanta of appropriate quantum fields. Although such fields, as classical entities, are not familiar to us, it is worth introducing them. The quantization of such fields will yield quanta, which will correspond to these elementary particles. In this way we build up a theory, known as *quantum field theory*, in which the fundamental entities are the quantum fields and not their quanta. The latter correspond to the known elementary particles of nature. Why bother? This is a way of incorporating in one theory the requirements of quantum mechanics and the special theory of relativity. (In the last 25 years or so particle physicists have discovered that there are other ways of incorporating the requirements of quantum mechanics and the special theory of relativity, and these ways gave rise to the so-called *quantum string and brane theories*, which may have the advantage over quantum field theory of allowing a consistent quantization of the field of gravity.)

It is worth pondering on the important features of quantum field theory. First the quantum fields are the primary objects, and particles come out of the theory automatically as the quanta of fields. The theory allows creation and annihilation of matter in the form of creation and annihilation of particles and antiparticles. The particles of a given field are all identical; they have the same mass and spin. The theory also demands that for every particle there exist an antiparticle, which may be identical with the particle itself. The antiparticle has to have the same mass as the particle itself, and in the case of a charged particle the antiparticle has to have opposite charge. Within the framework of the theory, also, the concept of force can be generalized to allow for interactions among particles that can lead to the formation of new particles.

All the above amazing features of the theory are in accordance with what we observe in nature. It is a fact, for instance, that all electrons are exactly identical in mass, charge and spin. Moreover, if we have enough energy at our disposal, we can create an electron and antielectron pair and we find that the newly created electron and antielectron are identical in mass, charge and spin with any electron and antielectron found anywhere else in the universe.

In classical Newtonian physics the number of particles present in a system is a fixed prescribed quantity, since particles are immutable quantities. In quantum field theory the particle number is an observable quantity, which can have various possible outcomes, owing to the ability of quantum fields to create and annihilate particles and antiparticles. This is very much in accordance with experiment. When two particles, say two protons, collide the outcome of the collision depends on the energy available. If the energy is small the two protons collide elastically and we end up with the elastic scattering process $\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p}$. No new

Summary of the salient features of quantum field theory

- Particles emerge as the quanta of quantum fields.
- All particles of the same species have identical masses.
- To every particle there is an antiparticle. Particle and antiparticle have equal mass and opposite charge, unless the particle is electrically neutral, in which case the particle is its own antiparticle.
- Generalization of the concept of force to allow for creation and annihilation of matter.

Table 1. Sample baryons.

Symbol	Name	Quark content	Electric charge	Spin
p	proton	u u d	1	$\frac{1}{2}$
$\bar{\mathrm{p}}$	antiproton	$\bar{\mathrm{u}}\,\bar{\mathrm{u}}\,\bar{\mathrm{d}}$	−1	$\frac{1}{2}$
n	neutron	u d d	0	$\frac{1}{2}$
$\bar{\mathrm{n}}$	antineutron	$\bar{\mathrm{u}}\,\bar{\mathrm{d}}\,\bar{\mathrm{d}}$	0	$\frac{1}{2}$
Λ	lamda	u d s	0	$\frac{1}{2}$

Table 2. Sample mesons.

Symbol	Name	Quark content	Electric charge	Spin
π^+	pi-plus	$\mathrm{u}\,\bar{\mathrm{d}}$	1	0
π^-	pi-minus	$\mathrm{d}\,\bar{\mathrm{u}}$	−1	0
π^0	pi-zero	$(\mathrm{u}\,\bar{\mathrm{u}} - \mathrm{d}\,\bar{\mathrm{d}})/\sqrt{2}$	0	0
K^+	K-plus	$\mathrm{u}\,\bar{\mathrm{s}}$	1	0
K^-	K-minus	$\mathrm{s}\,\bar{\mathrm{u}}$	−1	0
K^0	K-zero	$\mathrm{d}\,\bar{\mathrm{s}}$	0	0
$\bar{\mathrm{K}}^0$	K-zero bar	$\mathrm{s}\,\bar{\mathrm{d}}$	0	0

particles are created. At high energies, however, a proton–proton collision can be inelastic, leading to the formation of new particles. For example, $\mathrm{p}+\mathrm{p} \to \mathrm{p}+\mathrm{p}+\pi^0$, where π^0 is a new particle that came into existence as a result of the collision. The Newtonian concept of the force as a rate of change of momentum can never explain such a process.

Particles

All known matter of the universe appears to be made out of particles, which belong to one of three categories. These are *leptons*, *hadrons* and *gauge bosons* (some of the gauge bosons are massless).

The leptons are

$$\begin{pmatrix}\nu_\mathrm{e}\\ \mathrm{e}\end{pmatrix} \quad \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix} \quad \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$$

The letter e stands for the familiar electron. The greek letters μ and τ stand for the muon and tau particles. They are very much like the electron, but much more massive, and unlike the electron they are unstable. The symbols ν_e, ν_μ and ν_τ represent the three types of neutrinos, which are very elusive particles capable of travelling through the entire Earth without interacting with other particles. Electron-neutrinos are produced copiously in the cores of stars like our Sun. To all these leptons we have to add their antiparticles:

$$\begin{pmatrix}\bar{\nu}_\mathrm{e}\\ \bar{\mathrm{e}}\end{pmatrix} \quad \begin{pmatrix}\bar{\nu}_\mu\\ \bar{\mu}\end{pmatrix} \quad \begin{pmatrix}\bar{\nu}_\tau\\ \bar{\tau}\end{pmatrix}$$

A letter with a bar denotes the antiparticle of the particle corresponding to that letter. This is standard notation, but not universal. For example, sometimes one uses e^- for the electron and e^+ for its antiparticle, which is called the positron.

As far as we know, with the technology available at present, all the leptons are elementary particles. Hadrons, on the other hand, are not elementary. They are bound states of other elementary particles called *quarks* (Murray Gell-Mann chose the name ‘quarks’, which is a ‘nonsense’ word used by James Joyce in his novel *Finnegans Wake*). There are six quarks, which are grouped in pairs like the leptons:

$$\begin{pmatrix}\mathrm{u}\\ \mathrm{d}\end{pmatrix} \quad \begin{pmatrix}\mathrm{c}\\ \mathrm{s}\end{pmatrix} \quad \begin{pmatrix}\mathrm{t}\\ \mathrm{b}\end{pmatrix}$$

The letter u denotes the *up* quark, the letter d the *down* quark, the letter s stands for the *strange* quark, the letter c for the *charm* quark and the letters t and b for the *top* and *bottom* quarks respectively. Together with the six quarks we have, of course, their antiparticles denoted by $\bar{\mathrm{u}}$, $\bar{\mathrm{d}}$, $\bar{\mathrm{c}}$, $\bar{\mathrm{s}}$, $\bar{\mathrm{t}}$, $\bar{\mathrm{b}}$.

The hadrons are made out of quarks. Those hadrons that are bound states of three quarks (q q q), or three antiquarks ($\bar{\mathrm{q}}\,\bar{\mathrm{q}}\,\bar{\mathrm{q}}$), are called *baryons*, whereas those that are bound states of a quark and an antiquark ($\mathrm{q}\,\bar{\mathrm{q}}$) are called *mesons*. A sample of baryons and mesons is given in tables 1 and 2.

Table 3. Leptons and quarks in the Standard Model.

	Leptons (spin = $\frac{1}{2}$)			Quarks (spin = $\frac{1}{2}$)		
Generation		Flavour	Electric charge		Flavour	Electric charge
I	ν_e	electron-neutrino	0	u	up	2/3
	e	electron	-1	d	down	$-1/3$
II	ν_μ	muon-neutrino	0	c	charm	2/3
	μ	muon	-1	s	strange	$-1/3$
III	ν_τ	tau-neutrino	0	t	top	2/3
	τ	tau	-1	b	bottom	$-1/3$

Finally the class of gauge bosons includes *gluons* (eight of them), W^+, W^-, Z and γ, the photon.

Photons, as the particles of electromagnetic radiation, are very familiar. As free particles they have zero mass, they are electrically neutral and travel with the speed of light. As virtual particles, however, they are the avatars of electromagnetic interactions.

W^+, W^- and Z are the electrically charged (W^+, W^-) and neutral (Z) weak interaction gauge bosons. They are less familiar than the photons, but they are responsible for the decay of nuclei, which produces beta-radiation. As free particles they are very massive ($m_{W^+} = m_{W^-} \approx 80m_p$ and $m_Z \approx 90m_p$, where m_p is the mass of the proton) and highly unstable. These gauge bosons were produced as free particles, for the first time in the history of mankind, in the early 1980s using the LEP accelerator at CERN. (To create out of energy a bit of matter that has mass about 80 or 90 times the mass of the proton is an awesome business.) When they are exchanged as virtual particles, these gauge bosons give rise to weak interactions. Such interactions allow, for example, a free neutron to decay into a proton with the emission of an electron and its antineutrino.

The gluons are responsible for the strong forces that hold the quarks together in their bound states. They are massless and electrically neutral, like the photons, but they carry the so-called colour charge and, unlike photons, they can interact directly among themselves. In fact it possible through these self-interactions to create a kind of matter called *gluonium*.

The photon, Z and the eight gluons are their own antiparticles, whereas W^+ and W^- form a charge conjugate pair. If W^+ is called the particle, then W^- is the antiparticle and vice versa.

The Standard Model

The quantum field theory that can account for all the leptons, quarks, gauge bosons and their antiparticles that we have discussed above, including their interactions, is known as the *Standard Model*. Strictly speaking, this model incorporates a quantum field that does not correspond to any of the particles mentioned above. This is the so-called *Higgs field*. Consequently the Standard Model predicts the existence of another particle, which is called the Higgs particle. The theory demands that this is a massive, electrically neutral, spin-zero particle. So far it has not been discovered. When the new generation of particle accelerators (like the LHC (Large Hadron Collider) at CERN[2]) comes into operation, a number of experiments are planned to look for and hopefully establish the existence of this rather 'mysterious' particle. The Higgs field, through its interactions with the other fields of the Standard Model, is responsible for allowing the leptons and the quarks to have mass without breaking the important symmetry of the theory known as *gauge symmetry*. This mechanism for generating masses was developed by Peter Higgs.

Tables 3 and 4 summarize the fundamental particles of the Standard Model. To the particles in table 3 we must add the six antileptons and the six antiquarks. Note that leptons and quarks are subdivided into three distinct sets called *generations* or *families*. All stable matter is made out of particles of the first generation. This is because all second- and third-generation leptons and all hadrons made out of second- and third-generation quarks are unstable and quickly decay into stable first-generation particles.

The ingredients of the Standard Model are summarized in table 5. In this description the basic

[2] PPARC, *Frontiers*, Issue 20, Autumn 2004.

Table 4. Gauge bosons in the Standard Model.

Gauge bosons (spin = 1)	Electric charge	Interaction
g, gluons (eight)	0	Strong (conserve flavour)
W^+ W^- Z	+ 1 −1 0	Weak (flavour changing)
γ, photon	0	Electromagnetic (conserve flavour)

Table 5. The ingredients of the Standard Model.

Interaction	Strong	Weak	Electromagnetic
Matter fields	Quarks	Quarks, leptons and Higgs particles	Electrically charged particles
Gauge fields	Gluons	W^+, W^-, Z	Photon
Conserved quantities (not an exhaustive list)	Baryon number Electric charge	Baryon number Lepton number Electric charge	Baryon number Lepton number Electric charge
Representative effect	Stable proton (bound states of quarks) Stable nuclei Fusion, Fission	Unstable neutron Beta decay Pion decay Muon decay	Stable atoms and molecules Electromagnetic spectrum Chemical reactions

constituents of matter are quarks and leptons, and these constituents interact with each other through the exchange of gauge bosons.

Feynman diagrams

In quantum field theory the scattering or decay of particles is described in terms of transition amplitude. For a transition process $|\mathrm{i}\rangle \rightarrow |\mathrm{f}\rangle$, where $|\mathrm{i}\rangle$ describes the initial particles present and $|\mathrm{f}\rangle$ describes the particles present in the final state, the transition amplitude is formally written as $\langle \mathrm{f}|S|\mathrm{i}\rangle$, where S is an operator known as scattering matrix. Every process $|\mathrm{i}\rangle \rightarrow |\mathrm{f}\rangle$ is characterized by the so-called *cross-section*, which is a measure of the probability for this process to occur. The importance of the transition amplitude $\langle \mathrm{f}|S|\mathrm{i}\rangle$ lies in the fact that the cross-section is proportional to the square of the transition amplitude. (Strictly speaking, since the transition amplitude is a quantity described in terms of a complex number, the cross-section is proportional to the square of the modulus of this number.)

Exact calculations of $\langle \mathrm{f}|S|\mathrm{i}\rangle$ are not currently possible due to the mathematical complexities of the theory of interacting quantum fields. In some cases, however, it is possible to develop approximate procedures (known as perturbation theory methods), that allow approximate calculations of the transition amplitude. In such approximations the quantity $\langle \mathrm{f}|S|\mathrm{i}\rangle$ can be expanded as an infinite series of terms of diminishing importance. Thus,

$$\langle \mathrm{f}|S|\mathrm{i}\rangle = \langle \mathrm{f}|S^{(1)}|\mathrm{i}\rangle + \langle \mathrm{f}|S^{(2)}|\mathrm{i}\rangle + \langle \mathrm{f}|S^{(3)}|\mathrm{i}\rangle \dots .$$

This approximation is quantitatively useful only if the first few terms in the series expansion provide a good enough approximation. This is like expanding

$$(1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}x^2 + \dots$$

and then keeping the first two terms in the expansion. For small values of x, of course, $(1+x)^{-1/2}$ is very nearly $1 - x/2$.

All this is very formal and abstract. It turns out, however, that it is possible to represent the above expansion in terms of diagrams. The diagrams representing the various terms of the expansion are called *Feynman diagrams*. They were introduced by Richard Feynman, when he developed the theory of *Quantum Electrodynamics* (QED) for which he shared the 1965 Nobel Prize in Physics with Julian Schwinger and Sin-Itiro Tomonaga. The diagrams offer an intuitive way of visualizing transition amplitudes of processes that involve scattering and decay of particles.

Table 6.

(a)	Creation of antielectron + creation of electron + creation of photon	$0 \rightarrow \bar{e} + e + \gamma$
(b)	Creation of antielectron + creation of electron + annihilation of photon	$\gamma \rightarrow \bar{e} + e$
(c)	Creation of antielectron + annihilation of antielectron + creation of photon	$\bar{e} \rightarrow \bar{e} + \gamma$
(d)	Creation of antielectron + annihilation of antielectron + annihilation of photon	$\bar{e} + \gamma \rightarrow \bar{e}$
(e)	Annihilation of electron + creation of electron + creation of photon	$e \rightarrow e + \gamma$
(f)	Annihilation of electron + creation of electron + annihilation of photon	$e + \gamma \rightarrow e$
(g)	Annihilation of electron + annihilation of antielectron + creation of photon	$e + \bar{e} \rightarrow \gamma$
(h)	Annihilation of electron + annihilation of antielectron + annihilation of photon	$\bar{e} + e + \gamma \rightarrow 0$

How do Feynman diagrams come about? To answer this, one has to consider the basic interactions allowed by the Standard Model. It is easier first to consider the more familiar electromagnetic interactions.

Electromagnetic interactions

Let $\psi(x)$ be the quantum field associated with any one of the electrically charged leptons (e, μ, τ) or the six quarks (u, d, c, s, t, b). The x that appears in the bracket is an abbreviation for (t, x, y, z), where t stands for time and (x, y, z) are space co-ordinates. If $\psi_e(x)$ represents the quantum field associated with electrons, then $\psi_e(x)$ can create an electron or annihilate an antielectron at x. To satisfy the requirements of the special theory of relativity the Standard Model incorporates another field, denoted by $\bar{\Psi}_e(x)$. This field can create an antielectron or annihilate an electron at x. Since we are dealing with electromagnetic interactions we must also consider the gauge field associated with photons, denoted by $A(x)$.

The electromagnetic interactions come about by allowing the quantum fields $\psi(x)$, $\bar{\Psi}(x)$ and $A(x)$ to interact at any point x. This interaction is denoted formally as

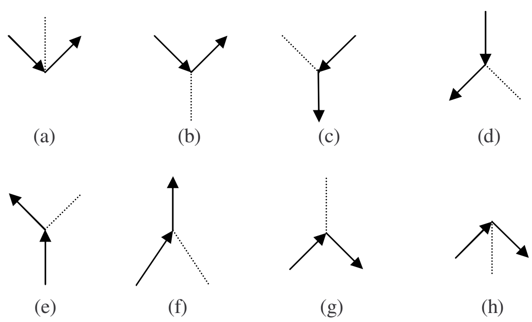
$$\bar{\Psi}(x)\psi(x)A(x).$$

Taking into account what each field can do at x, it is clear that we can have the potential processes listed in table 6.

Let us now introduce the following diagrammatic notation:

A dot, $\cdot$, represents the point x. An arrow $\rightarrow$ drawn upwards going into the dot represents annihilation of an electron. An arrow drawn upwards going away from the dot represents creation of an electron. An arrow drawn downwards away from the dot represents annihilation of an antielectron, whereas an arrow drawn downwards going into the dot represents creation of an antielectron. A dotted line drawn upwards going into the dot represents annihilation of a photon and going away from the dot represents creation of a photon[3].

Using the above notation the eight potential processes shown in table 6 can be represented diagrammatically as follows:

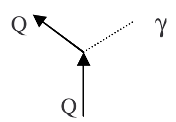


Starting with the transition $e \rightarrow e+\gamma$, it is possible to get the remaining seven transitions by applying two rules:

Rule 1: We can move a particle from one side of the transition to the other and replace it with its antiparticle.

Rule 2: We can replace all the particles of the transition with their antiparticles.

So the single diagram representing the transition $e \rightarrow e + \gamma$ subsumes all the potential transitions and can be taken to represent the basic electromagnetic interaction. In the Standard Model, therefore, the single diagram

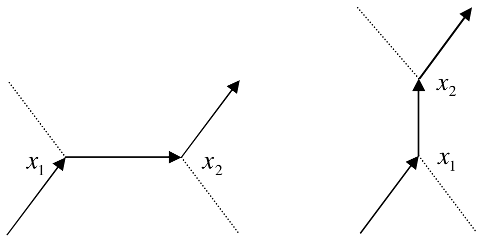


[3] The standard notation for a photon is a wavy line. We have adopted the dotted line because it is easier to draw.

can be taken to represent the basic electromagnetic interaction of any charged fermion Q with the photon. Here Q can be any one of the leptons (e, μ, τ) or any one of the quarks (u, d, c, s, t, b).

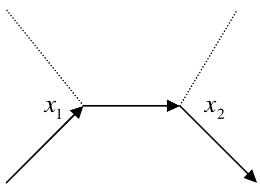
Building up diagrams from elementary processes

First consider the elementary process $e \rightarrow e + \gamma$ represented by the elementary diagram (e). Physically, an electron cannot convert to an electron and a photon. Energy–momentum conservation forbids this process. However, we can combine the two elementary diagrams (e) and (f) to build up the following diagrams with two vertices at x_1 and x_2:



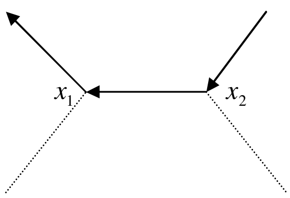
These two diagrams contribute to the physical process $e+\gamma \rightarrow e+\gamma$, which is known as Compton scattering and allows a photon to be scattered from a free electron.

Again if we combine the two elementary diagrams (e) and (g) we get the following diagram with two vertices:



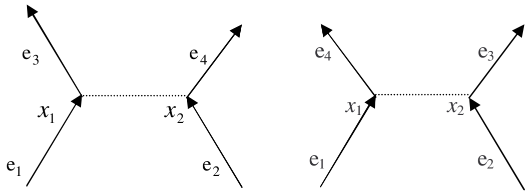
which contributes to the physical process of electron–positron annihilation, $e + \bar{e} \rightarrow \gamma + \gamma$.

As a third example we can combine the elementary diagrams (b) and (f) to build a new diagram with two vertices:



which contributes to the physical process of electron–positron creation, $\gamma + \gamma \rightarrow e + \bar{e}$.

As a final example of electromagnetic processes we combine the elementary diagrams (e) and (f) to produce

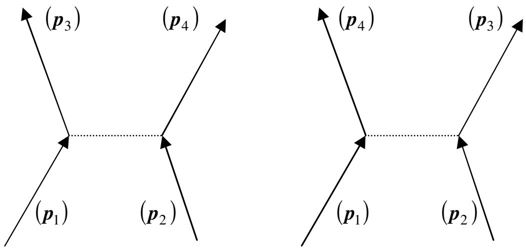


which are diagrams with two vertices contributing to the physical process of electron–electron scattering, $e + e \rightarrow e + e$.

In all the above diagrams the lines that appear at the bottom of the diagram (that is the lines that start at the bottom of the diagram and join the vertices) represent real particles present in the initial state $|i\rangle$ and the lines that appear at the top of the diagram (that is the lines that start at the vertices and are drawn upwards) represent real particles present in the final state $|f\rangle$. It is in this sense that the arrow of time runs from bottom to top. It must be emphasized, however, that in all Feynman diagrams there is **no time ordering of the vertices**. For example, in the Feynman diagram representing the electron–electron scattering it is incorrect to claim that a photon is first emitted at x_1 and absorbed later at x_2 or vice versa. The orientation, therefore, of the line joining x_1 to x_2 is of no significance. These internal lines, which join the vertices, represent *virtual* particles.

There are well-defined rules that allow calculations to be carried out to extract the scattering amplitude $\langle f|S^{(n)}|i\rangle$ from the diagram. According to these rules one has to integrate over all possible values of the space–time coordinates of the various vertices of the diagram. In the previous examples of Feynman diagrams, therefore, we have to integrate over (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2). This process of integration, basically, turns the time–space Feynman diagrams into energy–momentum diagrams. What is remarkable, also, is that the integration ensures that **energy and momentum are conserved at each vertex and overall** for the process represented by the diagram. The energy–momentum diagrams are by far the most convenient, because in scattering experiments the particles appearing in the initial and final states are labelled in terms of their momenta and energies.

In the case of electron–electron scattering, for example, we get



In such diagrams energy and momentum are conserved at every vertex of the diagram, leading to overall energy–momentum conservation. Also, for every external line that represents a real particle we have $E^2 = p^2c^2 + m^2c^4$, where E, p and m are the energy, momentum and mass of that particle. For internal lines, however, which represent virtual particles, $E^2 \neq p^2c^2 + m^2c^4$ (in the jargon of particle physicists the virtual particle is *off-mass shell*). This is what distinguishes virtual particles from real ones. 'Virtual particles' are not observable quantities, and in fact the concept of a virtual particle is useful only to the extent that it allows us to develop an intuitive understanding of interactions between particles. In the above Feynman diagram one can argue that the two electrons involved in the scattering e + e $\to$ e + e do exchange a virtual photon, and via this exchange they are able to feel each other's presence. Within the framework of quantum field theory, however, it is more accurate to think of particle interactions as a direct consequence of quantum field interactions giving rise to elementary vertex interactions.

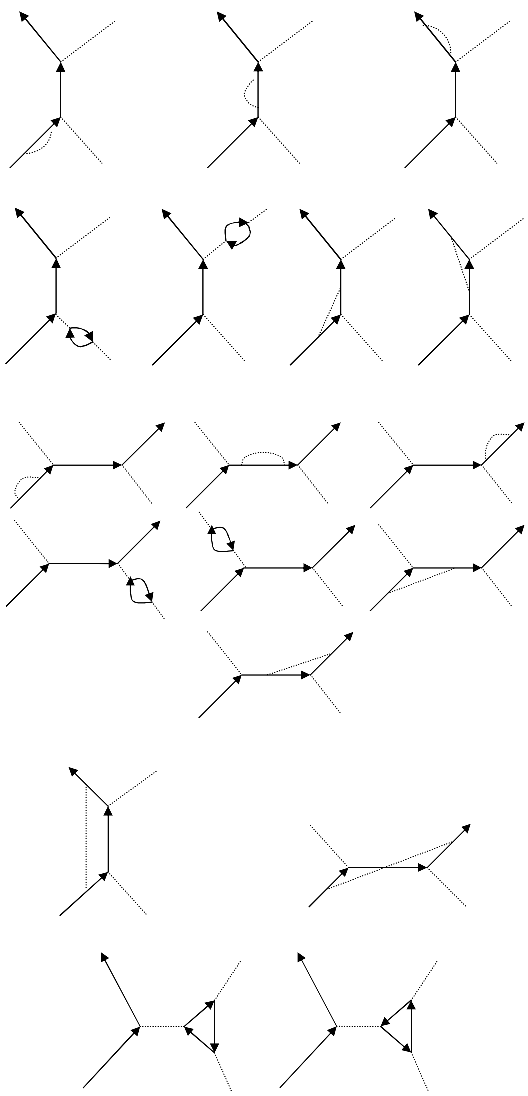
Hierarchy of diagrams

The Feynman diagrams with two vertices contributing to, say, the Compton scattering process e + γ $\to$ e + γ considered above are proportional to α, which is a dimensionless quantity given by

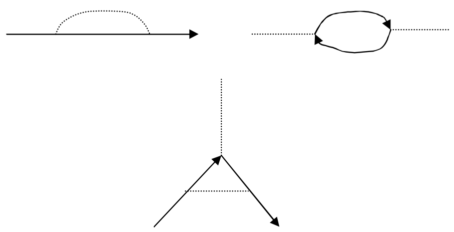
$$\alpha = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{\hbar c}$$

where ε_0 is the permittivity of free space, e is the electric charge of the proton, c is the speed of light and $\hbar$ is Planck's constant divided by 2π. A quick calculation shows that $\alpha \approx 1/137$.

Consider now the Feynman diagrams with four vertices that contribute to the same process of Compton scattering. These diagrams are shown below:



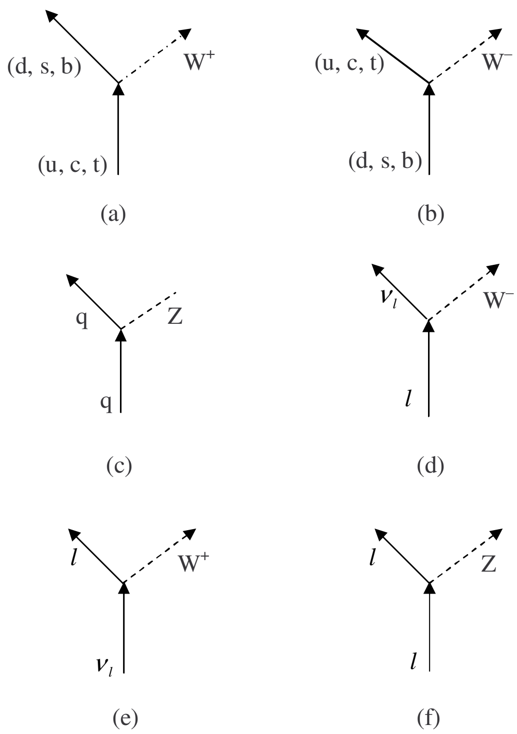
All the above diagrams are proportional to $\alpha^2 \approx (1/137)^2$. Their contribution to Compton scattering, therefore, is much smaller than the corresponding contribution from the diagrams with two vertices. To first approximation, therefore, their contribution can be ignored. However, if we want to confront the theory with accurate experimental results of electron–photon scattering, we must calculate the higher order correction from these diagrams with four vertices. It turns out that the computation of the Feynman diagrams, which have sub-diagrams of the form



produces infinities. These infinities, however, can be removed from the theory in a consistent way, through the process called *renormalization*. When this is done the diagrams with four vertices give a finite result, which is proportional to α^2. When this is added to the result from the diagrams with two vertices, which is of order α, we get the overall contribution, which agrees with the experimental results to a high degree of accuracy.

Weak interactions

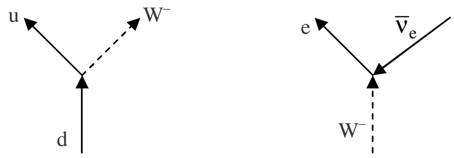
These can be described in terms of the basic Feynman diagrams



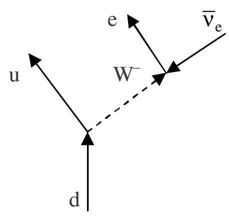
where q stands for any of the six quarks (u, d, c, s, t, b) and l stands for any one of the charged leptons e, μ, τ. As in the case of electromagnetic interactions these basic diagrams subsume a collection of fundamental processes obtained by applying the rule that we can move particles from down up and from up down and replace them with their antiparticles.

Weak interactions, unlike electromagnetic interactions, can produce flavour-changing transitions among the quarks. For example, an incoming d quark (see diagram (b) above) can change into an outgoing u quark by emitting a W^-. Also an incoming electron, muon or tau can change into an outgoing electron-neutrino, muon-neutrino or tau-neutrino respectively (see diagram (d) above) by emitting a W^+.

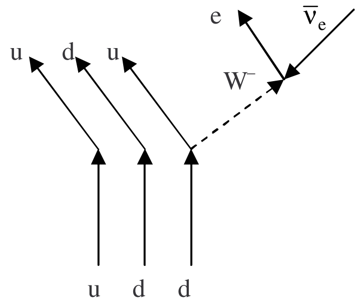
As in the previous section we can combine the above elementary processes to construct Feynman diagrams that contribute to various scattering and decay processes. For example, by combining



we get



Such a process allows a free neutron to decay into a proton, electron and electron-antineutrino as shown by the diagram below:



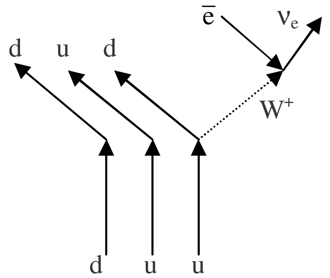
Bearing in mind that the bound state of the three quarks (u d d) represents a neutron and the bound state of the three quarks (u u d) represents a proton, we get

$$n \to p + e + \bar{\nu}_e. \qquad (1)$$

This decay also allows a neutron inside an unstable (neutron-rich) nucleus to decay, giving rise to beta-minus radiation.

Here is another example. By combining

we get



The above diagram leads to proton decay:

$$\mathrm{p} \rightarrow \mathrm{n} + \bar{\mathrm{e}} + \nu_{\mathrm{e}}. \tag{2}$$

This decay allows a proton inside an unstable (proton-rich) nucleus to decay, giving rise to beta-plus radiation.

Let us now consider the two processes represented by (1) and (2) and apply the two rules we have encountered earlier on. Starting with process (2) we can transport the antielectron from the right-hand side to the left-hand side and replace it with an electron to get

$$\mathrm{p} + \mathrm{e} \rightarrow \mathrm{n} + \nu_{\mathrm{e}}. \tag{3}$$

This process is known as *electron capture*, which allows a proton inside a nucleus to change into a neutron by capturing an electron from one of the inner shells of an atom. Starting with (3) now, and applying the rule that all particles can change into antiparticles, we get the following process:

$$\bar{\mathrm{p}} + \bar{\mathrm{e}} \rightarrow \bar{\mathrm{n}} + \bar{\nu}_{\mathrm{e}}. \tag{4}$$

By applying rule 1 to each particle the above process leads to

$$\mathrm{n} + \nu_{\mathrm{e}} \rightarrow \mathrm{p} + \mathrm{e} \tag{5}$$

which represents *neutrino–neutron* scattering.

Going back to (2) and transporting the electron-neutrino to the right-hand side and replacing it with an antineutrino, we get

$$\mathrm{p} + \bar{\nu}_{\mathrm{e}} \rightarrow \mathrm{n} + \bar{\mathrm{e}}. \tag{6}$$

The above reaction was studied by Cowan and Reines in 1958 and 1959 using a reactor as a source of antineutrinos. It was the first experiment in which antineutrinos were observed directly. In this experiment a flux of antineutrinos was directed towards a target of water containing about 10^{28} protons in which some $CdCl_2$ was dissolved. The antielectron produced when this process takes place annihilates quickly with an electron, producing two gamma rays. The recoiling neutron slows down through collisions with protons and is then captured in the Cd nuclei, producing gamma rays. So the characteristic signature of the production of two simultaneous gamma rays followed by another gamma ray, a few microseconds later, can be taken as direct evidence for the existence of antineutrinos. Pauli predicted the existence of the antineutrino in the early 1930s, but it took almost 30 years before it was confirmed experimentally.

Strong interactions

The strong interactions are responsible for the forces holding the quarks together to form baryons and mesons. To understand how these forces come about we have to endow the quarks not only with electric charge, but also with another kind of charge called '*colour*'. The colour charge is associated with the strong forces in the same way as the electric charge is associated with the electromagnetic forces. The colour charge, of course, has nothing to do with the colours of the visible spectrum; one has to think of it as another attribute of the quarks. The leptons have no colour, so leptons cannot participate in strong interactions because they are not susceptible to the strong force.

The particles that mediate strong interactions are the gluons. They are analogous to the photon, which is the avatar of electromagnetic interactions. However, whereas the photon has no electric charge, some of the gluons do carry colour charge.

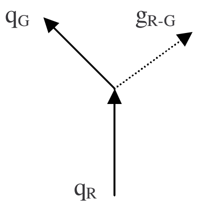
To take into account the colour charge the quarks must carry an extra label, to represent the colour. Each flavour of quark comes in three colours. We can have, for example, a red up quark,

u_R, or a green up quark, u_G, or a blue up quark, u_B. A red up quark can emit a gluon and turn into a blue up quark. The emission of the gluon leaves the quark type or flavour the same. The strong interactions, therefore, conserve flavour. There are altogether eight gluons. Six of them carry colour and two of them are colour neutral. The interactions between the quarks and gluons can be summarized as follows:

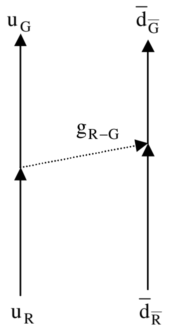
	q_R	q_G	q_B
q_R	$g_1 + g_2$	g_{R-G}	g_{R-B}
q_G	g_{G-R}	$g_1 + g_2$	g_{G-B}
q_B	g_{B-R}	g_{B-G}	$g_1 + g_2$

Any one of the quark colours in the first column can be transferred into any one of the quark colours in the first row; the transition is mediated by the gluon specified at the intersection of the appropriate column and row. A red quark can emit a g_{R-G} gluon to become a green quark. The g_1 and g_2 are the two colour-neutral gluons.

We can represent the above interaction through elementary diagrams. For example, the red quark turning into a green quark can be represented by



where q stands for any one of the six quark flavours, u, d, c, s, t and b. By combining such elementary diagrams we can create a Feynman diagram, like the one shown below:



This Feynman diagram shows how the strong force arising from the gluon exchange can keep an up and an anti-down quark together to form a bound state, which is the meson. At one vertex of this diagram the red quark changes into a green quark and emits a g_{R-G} gluon. Using advanced mathematics it can be shown that the colour of this gluon is equivalent to red + anti-green. This emitted gluon is absorbed at the other vertex of the diagram. At that vertex the red of the gluon and the anti-red of the incoming quark get annihilated, leaving an outgoing quark with anti-green colour.

Conclusion

In the above presentation of Feynman diagrams we start with the elementary diagrams, which represent diagrammatically the local interactions of quantum fields, and we proceed to build Feynman diagrams representing physical processes by combining these elementary diagrams. There is an excellent website with interactive features that allows the use of elementary diagrams to unfold a hadron decay by building up the relevant Feynman diagram step by step. To get to this site visit www.cpepweb.org/particles.html, click on 'The Particle Adventure' and then go to 'The Fireworks of Particles'.

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Further reading

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Michael Daniel is Head of Physics at King Edward's School, Birmingham. He has carried out research in theoretical elementary particle physics at several European universities, CERN and the Rutherford Appleton Laboratory. He taught physics at the King's School in Canterbury and was the Head of Science and Physics at St Paul's Girls School, London.